$$\frac{1}{4.96}=y-\frac{100}{96}y^2+\frac{35}{96}y^3-\frac{5}{96}y^4+\frac{1}{4.96}y^5,$$
$$-\frac{1}{96}=y-\frac{40}{96}y^2-\frac{20}{96}y^3+\frac{10}{96}y^4-\frac{1}{96}y^5,$$
$$+\frac{1}{64}=y+0\ y^2-\frac{20}{64}y^3+0\ y^4+\frac{1}{64}y^5,$$
$$-\frac{1}{96}=y+\frac{40}{96}y^2-\frac{20}{96}y^3-\frac{10}{96}y^4-\frac{1}{96}y^5,$$
$$\frac{1}{4.96}=y+\frac{100}{96}y^2+\frac{35}{96}y^3+\frac{5}{96}y^4+\frac{1}{4.96}y^5.$$

If we now develop each of these equations by (R) and add to the results the corresponding integers, 2, 4, 6, 8 and 10, we shall have the five roots of the equation. Thus,

$$x_1 = 2+\frac{1}{4.96}+\frac{100}{4^2.96^3}+\frac{16640}{4^3.96^5}+\ldots= 2.002611,$$
$$x_2 = 4-\frac{1}{96}+\frac{40}{96^3}-\frac{5120}{96^5}+\ldots= 3.989591,$$
$$x_3 = 6+\frac{1}{64}+\frac{20}{64^4}+\frac{1136}{64^7}+\ldots= 6.015626,$$
$$x_4 = 8-\frac{1}{96}-\frac{40}{96^3}-\frac{5120}{96^5}-\ldots= 7.989568,$$
$$x_5 = 10+\frac{1}{4.96}-\frac{100}{4^2.96^3}+\frac{16640}{4^3.96^3}-\ldots= 10.002597.$$

In like manner all irrational roots may be found by the rule expressed by (R) which may therefore be called the Root Theorem.

GEOMETRICAL DETERMINATION OF THE AREA OF THE PARABOLA.

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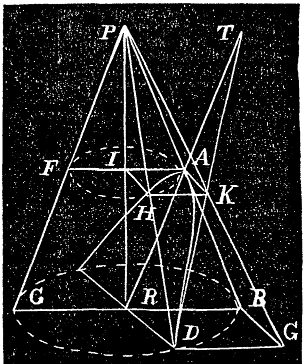


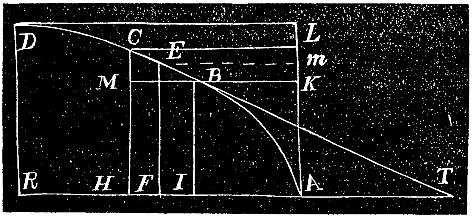
LET AR represent the principal axis of a parabola, so placed as to form one of the equal sides of an isosceles triangle ABR, with the base BR, equal to the corresponding ordinate of the given parabola, and at R erect a perpendicular RP to intersect AB produced in P.

Suppose the cone $PBDC$ completed by revolving PBR about PR; then will CPB, also, be isosceles, and similar to RAB, hence the line AR is parallel to PC, and AB is half of PB.

Along the line AR pass a plane with its cutt'g

edge at right angles with CB, and the section produced, AED, is the given parabola, RB being equal to RD. Through A pass a plane parallel to the base CBD cutting the circle AHF. Since PIA and PRB are similar triangles, $IA = \frac{1}{2}RB$.

Suppose the conical surface to be divided into an infinite number of triangles, as Prs which is supposed to terminate at the point D on the curve, with its base parallel to CB. On RB and RD form the square $RBGD$, and on AI and IH, radii at right angles, form the square $IAKH$, and complete the prismoid $HKGDRBIA$. The face $HKDG$ will be an expansion of the plane Prs, and the plane that cuts the parabola must also cut the prismoid from the corner K to the corner D, hence the line that is tangent at the point D of the parabola will pass through K. Prolong the axis RA to meet DK produced in T. Because AK $(= AI)$ by construct'n is parallel to RD $(= RB)$, and because it has been shown that $IA = \frac{1}{2}RB$, $\therefore AK = \frac{1}{2}RD$; $\therefore RA = \frac{1}{2}RT$, or the subtang't is bisected at A. The same relation can be shown for the tang't of any other point of the curve by erecting a cone in like manner on its coordinates, and demonstrating as above. When the ordinate exceeds the abscissa a cone with elliptic base must be used.



Let the parabola $ABCD$ (pr'l axis AR) be divided into an infinite number of parts, BC being one of these parts, and BI and CH ordinates at B and C, resp'y.

From E, the middle point of BC, let fall the perpendic'lr EF, and draw the tangent ET, then is $FT = 2FA$. On DR and RA complete the parallelogram $DRAL$ and draw CL, Em and MBK parallel to RA; then is $Lm = mK$.

The triangles CMB and EFT are similar, hence $CM : MB :: EF : FT$;

$$\therefore CM \times FT = MB \times EF. \qquad (1)$$

The trapezoid $CBHI$ is equal to $MB \times EF$ [by (1)] $= CM \times FT$. (2)

The trapezoid $CBKL$ is equal to $CM \times Em$, or since $Em = FA = \frac{1}{2}FT$,

$$\therefore CBKL = CM \times \tfrac{1}{2}FT. \qquad (3)$$

Hence, from (2) and (3), the trapezoid $CBHI = 2CBLK$. And as the same relation holds for all similar trapezoids drawn in the parallelogram $DRAL$, it follows that the area of the parabola is two thirds of its circumscribed parallelogram.